Uncertainty of Standard Penetration Test Measurements and its Effect on Geotechnical Design (Case Study)

Incertitude des mesures standard de test de pénétration et son effet sur la conception géotechnique (étude de cas)

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ABSTRACT

In all construction projects, sufficient information or data is required for a successful design. The site investigation phase of any geotechnical design plays a vital role to provide the geotechnical engineer by the most appropriate data to ensure that the design data is representative of the investigated soil. In geotechnical engineering practice, uncertainties of the field measurement are due to three main sources; inherent soil variability, measurement error and transformation model uncertainty or statistical uncertainty. Measurement uncertainty is divided into two categories: systematic and random errors. The main goal of this research is to identify the effect of the uncertainty of the standard penetration test results on the design and project construction. To achieve the research goal settlement and bearing capacity were calculated for a large project in Egypt, which consists of a 48 buildings with different structural systems, loads, and foundation systems and levels, as a case study. The site investigation for this project was taken in three stages pretender, during tender and post tender stage. A major variation in the field test results observed which affect the results of settlement and bearing capacity analysis. Three buildings were chosen for detailed study, as an example. For each building a comparison between the results has been made using the parameters of each stage once and for the average of the three stages parameters another time.

Keywords: uncertainty, standard penetration test, settlement, bearing capacity.

1 INTRODUCTION

1.1 Uncertainty in Geotechnical Engineering

"Certainty" refers to situations in which the outcome of an event or the value of a parameter is known with unit probability. Conversely, uncertainty occurs when a collection of values associated with respective uncertain "states of nature" occur with strictly non-negative probabilities for at least two different possible values; the simplest examples are tossing a coin or rolling a die. Uncertainty analysis is an emerging approach that uses estimation and simulation techniques to consider the variability of available data and to estimate the frequency with which values of interest are likely to be exceeded. While it has not yet been widely applied in geotechnical engineering practice, this approach offers insight into existing data for heterogeneous geotechnical systems (Jones et al. 2002).
Vanmarcke (1977a, 1977b) suggested that three main sources of uncertainty exist in the estimation of suitable soil properties. These are due to inherent soil variability, statistical uncertainty due to limited sampling, and measurement uncertainties due to associated geotechnical testing errors. Filippas et al. (1988) also categorized uncertainties in a geotechnical system into three main components: inherent soil variability, measurement error, and transformation model uncertainty. In addition, Kulhawy (1992) suggested statistical uncertainty, or sampling error, as introduced by Vanmarcke (1977a, 1977b), which results from limited information about the site. This component of uncertainty can be included with measurement error and is minimized through additional sampling (Vanmarcke 1977a, Vanmarcke 1977b, Phoon et al. 1995). Whitman (2000) adopted a simpler explanation, where the uncertainties due to soil variability and random testing errors contribute to data scatter, while the statistical uncertainty and bias in testing error contribute to systematic errors. Kulhawy and Phoon (2002) have also indicated that soil variability and measurement error have an impact on data scatter which indicates the stages at which each source of uncertainty affects the estimation of a soil property.
A slightly different approach to separating the sources of uncertainty has been adopted by Baecher and Christian (2003), who considered the sources to be: natural variability, knowledge uncertainty, and decision model uncertainty.
Essentially the first two sources identified by Baecher and Christian (2003) are equivalent to the sources identified by Kulhawy (1992), where natural and inherent soil variability are the same, and uncertainties due to measurement and transformation model error are equivalent to knowledge uncertainty. Furthermore, Baecher and Christian (2003) categorized knowledge uncertainty into effects dealing with site characterization, model and parameter uncertainty. Site characterization uncertainty accounts for both measurement errors and statistical uncertainty, as described by Filippas et al. (1988) and Kulhawy (1992), respectively, while model and parameter uncertainty matches well with transformation model error. The additional source of uncertainty identified by Baecher and Christian (2003), due to decision models, is a function of the decisions an engineer or client makes regarding conservatism, as well as effects during construction. Such uncertainties are usually due to economic and temporal considerations.
The following sections identify the three sources of uncertainty defined by Filippas et al. (1988), as well as the statistical uncertainty discussed by Kulhawy (1992). Some evidence regarding the magnitude of each source of uncertainty has been quantified. However, in general, the discussion in the following sections deals with the manner in which the uncertainties are best quantified.

1.1.1 *Inherent Soil Variability*

Unlike many civil engineering media, soils are inherently variable, where properties may be significantly different from one location to another. Even when soils are considered reasonably homogeneous, soil properties exhibit considerable variability (Vanmarcke 1977a). This variability is due to the complex and varied physical phenomena experienced during their formation (Jaksa 1995). Variability between soil properties is called spatial variability and has recently been modeled as a random variable (Spry et al. 1988).

1.1.2 *Statistical Uncertainty*

The statistical uncertainties associated with a geotechnical model are a result of limited sampling that may not provide an accurate representation of the underlying conditions. Filippas et al. (1988) defined the statistical uncertainty for a set of uncorrelated samples as the variance in the estimate of the mean.

1.1.3 *Measurement Error*

Measurement errors arise from the inability of geotechnical tests to accurately estimate the soil properties being tested. Sources of measurement error can be separated into 2 categories: systematic and random (Filippas et al. 1988). Random testing effects are inherent to the test type but cannot be attributed to the spatial variability of soil properties. The effects are generally considered to have zero mean and influence the results of the soil properties equally above and below the mean (Baecher 1979, Snedecor and Cochran 1980). Filippas et al. (1988) suggested the best way to evaluate random testing effects is by undertaking several tests under essentially identical conditions. Systematic errors consistently under or overestimate the property and are generally due to operator and procedural effects and inadequacies with the equipment (Jaksa et al. 1997). Lumb (1974) considered such errors as a bias.

1.1.4 *Transformation Model Uncertainty*

The results of common geotechnical in situ tests do not typically provide applicable soil properties that are useful for design relationships (Phoon and Kulhawy 1999b). Rather, the raw test results are processed using a transformation model into a suitable design parameter. Such models are obtained empirically through back substitution or calibration. Accordingly, a degree of uncertainty is added to the estimation of the design parameter. Phoon and Kulhawy (1999b) further stated that uncertainty still exists if the transformation is based on a theoretical relationship because of idealizations and simplifications in the theory. Therefore, it is important to consider the uncertainties due to transformation model error. Reliability methods have been increasingly used in geotechnical engineering, as part of site characterization (e.g. Whitman, 2000), in slope stability problems (e.g. Duncan, 2000), and in foundation engineering design (Phoon et al. 2003).

1.2 *Uncertainty of SPT Data*

A geotechnical problem in which the SPT is particularly popular is the design of shallow foundations. The Standard Penetration Test (SPT) is one of the most popular tools for geotechnical characterization of a site primarily due to its simplicity and economy. The Standard Penetration Test was first "standard-ized" by the American Society for Testing Materials (ASTM) in 1958 (designation D1586-58). It was soon recognized that the test was not a reliable source of information (e.g. Tavenas (1971), Fletcher (1965)). To increase the reliability of the in-situ test, the method was further standardized with the most recent update in 1999 (ASTM D1586-99). This update addresses many of the uncertainties involved and provides guidelines for the performance of the SPT. Further standardization of the method was made to improve the use of the SPT in evaluating the liquefaction potential by the ASTM D6066-96 standard (Zekkos et al.2004). Another extensive literature review was made to evaluate sources of uncertainty in the performance of the SPT, based on the previous work of many researchers (Kulhawy and Mayne (1990), Schmertmann (1975), Barton (1990), ASTM D6066-96, Youd et al. (2001), Kulhawy and Trautmann (1996)). Kulhawy and Trautmann (1996) reviewed the various sources of uncertainty and suggested that the total uncertainty in the N-value is for the best-case scenario on the order of 14% and for the worst-case scenario about 100%. Zekkos et. al (2004) mentioned that the total 27sources of uncertainty and bias could be summarizes as following:

A. Sources depending on encountered soil
- Vertical Stress
- Mineralogy
- Coarse gravel or cobbles in soil
- Horizontal stress
- Geologically aged sand deposits

B. Sources due to presence of water
- Pore pressure generation
- Moisture-sensitive behavior of geologically aged sands

C. Reducible sources related to equipment and its maintenance
- Hammer efficiency
- Borehole diameter
- Sampler
- Rod Length
- Lack of hammer free fall because of ungreased sheaves, new stiff rope for lifting weight No Use of bent drill rods
- Bottom vs. side discharge bits
- Hammer weight inaccurate
- Type of drilling equipment

D. Reducible sources with careful site investigation procedure
- Inadequate cleaning of hole
- Inadequate head of water in the borehole
- Careless measurement of hammer drop
- Sampler driven above bottom of casing
- More than two turns on cathead
- Hammer strikes drill rod collar eccentrically
- Incomplete release of rope in each drop
- Tightness of connections
- Careless blow count

E. Irreducible sources in investigation procedure
- Human factor
- Weather and site conditions

More details about the influence of the sources of uncertainty could be found in Geoengineer website (www.geoengineer.org).

1.3 *Case Study Information*

The case study is a combined-cycle power plant project in Giza North Government, Egypt. The project site is located about 22 km north of Qanater city. The site area is about 290,000 m2. The site is an agricultural farmland and planted with several trees. There were several ditches and canals existed on site and several parts of it were covered with water from irrigation. The project is consists of a 48 buildings with different structural systems. The main units are intake structure, pump house, two large diameter gasoline tanks, power block area, seal well structure, discharge culvert, discharge and storage areas, fire fighting station, circulating water pipe network, and several ancillary buildings including warehouse, workshop, construction building, office building, security offices, fence, and guard towers. The buildings have different foundation systems, with or without basement.

The site investigation for this project had been taken in three stages pre-tender, during-tender and post-tender stage. The number of boreholes were (124) with depths varying from 25.0 m and 50.0 m in the pretender stage, (22) boreholes with depths varying from 10.0 m and 35.0 m in the during-tender stage, and (19) boreholes with depths varying from 15.0 m and 20.0 m in the post-tender stage. The depths were measured from the existing ground at the time of the investigation.

2 METHODOLOGY

Any civil engineering system can have different modes of failure. Zekkos et. al. (2004) mentioned that the engineer needs to design with safety and economy in mind, and the foundation system will "fail" (i.e. not perform satisfactorily) if any of the two following criteria is violated.

1. Bearing Capacity Failure: caused by the exceedance of the shear strength of soil; this mode of failure is abrupt and catastrophic.
2. Excessive Settlement: defined as the exceedance of a specified maximum acceptable amount of settlement.

Three buildings were chosen for detailed study against these two types of failure. For each building a comparison between the results has been made using the parameters derived from SPT results of each stage individually once and for the average of the three stages parameters another time. The first building is the warehouse which consists of three floors and one basement. The foundation system of this building is a reinforcement concrete raft with dimensions of 42x42 m. The second building that studied in this paper is the workshop building with two floors and without basement. The foundation system of this building is transferred to a reinforcement concrete raft with dimensions of 36x40 m. The last chosen building is the guard dormitory which consists of two floors without basement and its foundation system is reinforced isolated footings.

The site stratification includes sand and clay layers. The soil general stratigraphy in the project site is consists of top agricultural layer with thickness less than 0.6 m from the ground surface. The top layer is followed by sand layer. This layer in some parts is interlayered with clay layer with varying thickness but in general less than 1.50 m, after the sand layer there are thick clay lowered by sand layer until the end of boring. An average soil profile for the project site is presented in Fig. (1). Since the purpose of the paper is to evaluate the effect of SPT test on the geotechnical design; an average clay properties of the have been unified. The clay thickness and properties have been estimated using the tests results from the three stages.

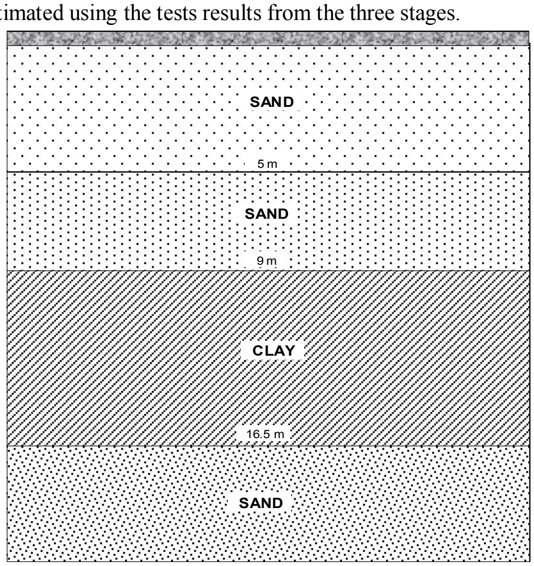

Fig. 1. Average soil profile

The variation of undrained shear strength based on measured values of pocket penetrometer, vane shear and unconfined compressive strength tests versus depth is shown in Fig. (xx). Based on the shown values, the undrained shear strength (cu) based on the pocket penetrometer is in the range between 0.10 kg/cm2 and 2.13 kg/cm2 with an average of 1.185 kg/cm2, while that based on unconfined compressive strength is in the range 0.65 kg/cm2 and 2.1 kg/cm2 with an average of 1.33 kg/cm2. The undrained shear strength based on laboratory Vane Shear test results is in the range of 0.60 kg/cm2 and 2.0 kg/cm2 with an average of 1.22 kg/cm2.

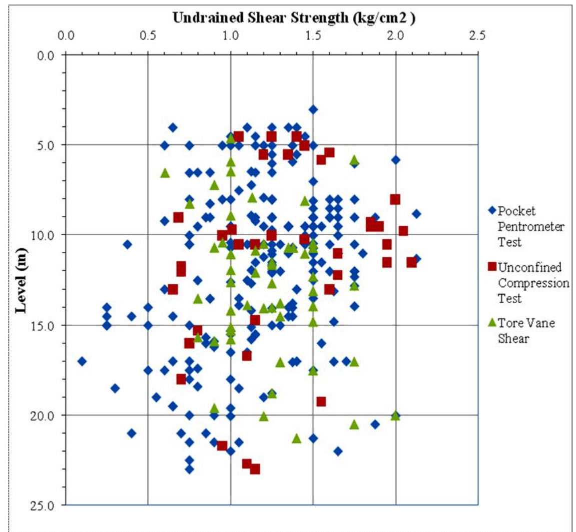

Fig. 2. Variation of undrained shear strength with depth

Utilizing the results of oedometer tests carried out in the site, the value of c_c after ignoring unreasonable can be considered in the range of 0.129 to 0.543 with an average of 0.264. Results of the oedometer tests resulted in ratio between c_r and c_c of (0.10 to 0.15). Utilizing the results of oedometer tests carried out in the site, the OCR of the clay can be considered 1.62. The voids ratio assuming fully saturated clay can be considered in the range of 0.555 to 1.570 with an average of 0.836.

3 ANALYSIS AND RESULTS

The whole project site has been studied in three stages of investigation in the project life. Those stages are pre-tender, during-tender and post-tender stage. The general soil profile was divided to three main sand layers depending on SPT results and one clay layer as shown in Fig.1.To know the consistency of the row SPT data results for the three stages, normal distribution curves were drawn for each stage as shown in Figs. 1,2, and 3. If we adopt the pre-tender stage as base for the comparison the results will be as following.

For the first sand layer, the mean SPT value of the during-tender is 69.5% more than the pre-tender SPT mean value and the mean SPT value of the post-tender is 36.5% less. The standard deviation of the during-tender is 6.6% less than the pre-tender standard deviation and the standard deviation of the post-tender is 44.0% less.

For the second sand layer, the mean SPT value of the during-tender is 32.1% more than the pre-tender SPT mean value and the mean SPT value of the post- tender is 37.1% less. The standard deviation of the during-tender is 90.9% less than the pre-tender standard deviation and the standard deviation of the post-tender is 24.5% less.

For the third sand layer, the mean SPT value of the during-tender is 2.5% more than the pre-tender SPT mean value and the mean SPT value of the post- tender is 37.9% less. The standard deviation of the during-tender is 99.9% less than the pre-tender standard deviation and the standard deviation of the post-tender is 86.3% more

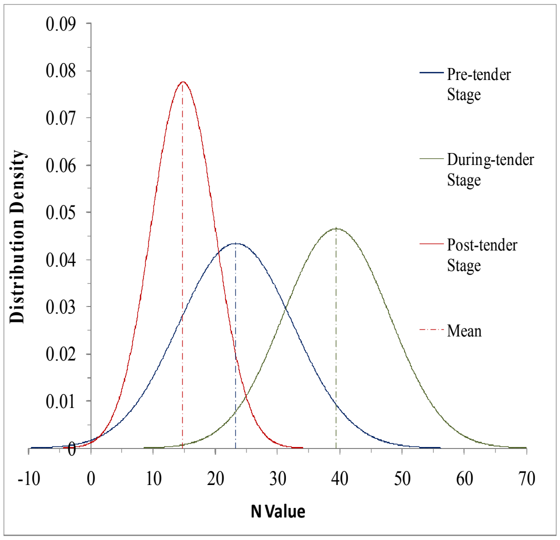

Fig. 3. SPT data normal distribution curve for first sand layer

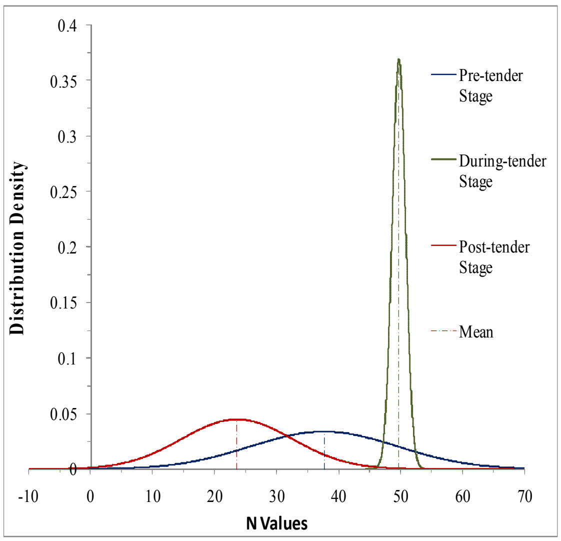

Fig. 4. SPT data normal distribution curve for second sand layer

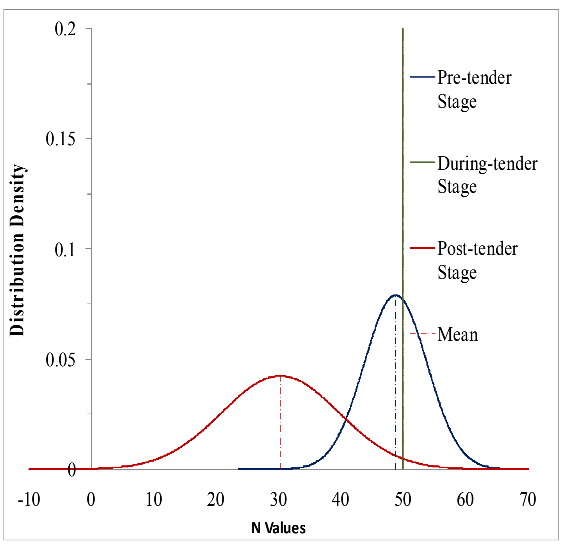

Fig. 5. SPT data normal distribution curve for third sand layer

For sand layers the variation of the SPT test results along the borehole depth for the three stages is shown in the Fig. (6). According to the SPT values an average friction angle (ø) was chosen for the three sand layers for every boreholes stage. Deformation modulus for the sand layers has been chosen based on different methods, Egyptian Code (2007), D'Appolonia andBrissette (1970), and Burland and Burbidge (1985). Based on these methods the used deformation modulus values for the three sand layers are shown in table 1.

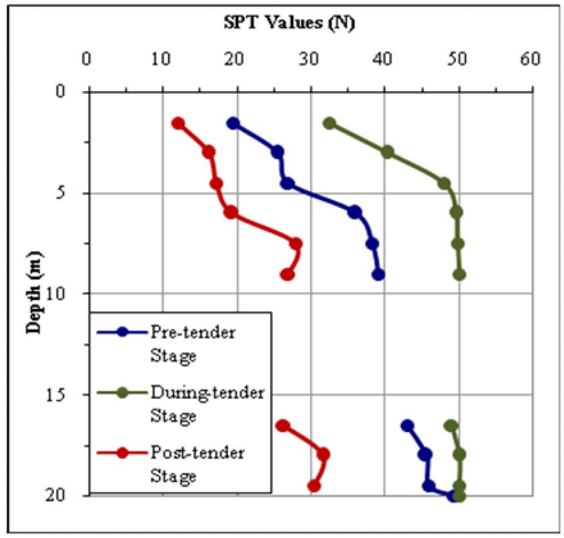

Fig. 6. SPT data normal distribution curve for third sand layer

3.1 Bearing Capacity results

For a granular, non-cohesive material, the ultimate bearing capacity q_{ult} of a saturated soil having a buoyant unit weight γ_b, under a foundation of width B, founded at a depth D, is often taken as (also known as the Terzaghi Bearing Capacity equation):

$$q_{ult} = \gamma_b \cdot D \cdot N_q + \frac{1}{2} \cdot \gamma_b \cdot B \cdot N_\gamma$$

Where Nq and Nγ are the bearing capacity factors, and these factors are dependent on the internal friction angel (ϕ), which derived based on the value of the SPT (N) number.

The bearing capacity has been calculated for the buildings. The results show that the bearing capacity has variation in the safe margin for the three stages which does not affect the design procedure.

3.2 Settlement Analysis Results

The calculations were carried out using in-house Software for settlement calculation (SETMAX) that was developed by Prof. Maximovic for Hamza Associates. This program computes the value of the settlement of the point on the surface of the layered system by integrating strains at depth. For defining the "type of soil compressibility", two of the options available in the program were used and which are defined as follows:

- Type 1. (MD model)

MD indicates that the constrained modulus independent of the stress change is defined. This model is used for determining the compression in the sandy layers simply by:

$$\varepsilon_z = \frac{\Delta\sigma'_z}{M_v}$$

- Type 2. (CL model)

CL model is the conventional Clay model. Parameters are derived from the conventional consolidation test interpreting results in terms of relationship between the void ratio and the logarithm of the normal effective stress. The link in this bilinear relationship reflects the over-consolidation stress. The following parameters are utilized in computing the settlement:

c_c: compression index valid for the stress level above pre-consolidation stress.

c_r: re-compression index applicable for the stress range lower than the pre-consolidation stress.

e_o : initial soil void ratio

P_o : initial vertical effective stress

P_c : Pre-consolidation stress.

The strain will be computed by one of the following expression:

If $p_o + \Delta\sigma'_z < p_c \longrightarrow \varepsilon_z = \frac{C_r}{(1+e_o)} \log\left(\frac{p_o + \Delta\sigma'_z}{p_o}\right)$

If $p_o + \Delta\sigma'_z > p_c \longrightarrow \varepsilon_z = \frac{C_r}{(1+e_o)} \log\left(\frac{p_c}{p_o}\right) + \frac{C_c}{(1+e_o)} \log\left(\frac{p_o + \Delta\sigma'_z}{p_c}\right)$

Strains are computed in each slice and integrated numerically for the whole depth of the given layer or sequence of layers. Settlements under the different points of flexible foundations are calculated where as for rigid footings the settlement is calculated by using the equivalent Kany's points. Kany's points are four points centrally symmetrical to the footing center. If the single footing is considered, settlement for the four points will be the same; however, in the case of eccentricity or influence of the neighboring loads, the settlement at the different points has been considered to estimate the rotation of the footing.

In the calculation of settlement of the different structures we considered the soil formation also the effect of the weight of the surrounding backfill and close structures. The full settlement (short and long term) under the structure load and surrounding loads at different points under the raft of each structure are computed.

Table 2 present the settlement results for the chosen buildings by different calculation methods. The calculated settlement considered thickness of influenced soil to depth corresponding to stress increase of 10 percent of the surface load and to a depth that corresponds to stress increase of the 10 percent of the overburden pressure. For the purpose of comparison, the settlements for each stage were compared to the allowable limits. For the Egyptian code calculation method, the allowable settlement has been defined as mentioned in ECP 202-2001, while for the other calculation methods, the allowable settlement numbers have been taken according to MacDonald and Skempton (1995).

If we adopt the pre-tender stage as base for the comparison the results will be as following.

For the first building, according to Egyptian code and Burland and Burbidge calculation methods the settlement value for the during-tender is about 14.0% less and it is about 43.0% more for the post-tender, D'Appolonia calculation method shows that the settlement value for the during-tender is 10.0% less and the settlement value of the post-tender is 21.0% more.

Table 1. General soil deformation modulus

Soil layers	Deformation modulus (MPa)								
	Egyptian Code			D'Appolonia & Brissette			Burland & Burbidge		
	Pre-tender	During-tender	Post-tender	Pre-tender	During-tender	Post-tender	Pre-tender	During-tender	Post-tender
Sand 1	16	28	10	31	48	28	48	101	26
Sand 2	26	35	17	48	58	31	94	140	49
Sand 3	32	35	21	55	60	41	125	140	69

For the second building, according to Egyptian code and Burland and Burbidge calculation methods the settlement value for the during-tender is about 17.0% less and it is about 40.0% more for the post-tender, D'Appolonia calculation method shows that the settlement value for the during-tender is 12.0% less and the settlement value of the post-tender is 17.0% more.

For the third building, according to Egyptian code and Burland and Burbidge calculation methods the settlement value for the during-tender is about 19.0% less and it is about 37.0% more for the post-tender, D'Appolonia calculation method shows that the settlement value for the during-tender is 15.0% less and the settlement value of the post-tender is 15.0% more.

Table 2. Summary of settlement analysis results for the chosen buildings

Method	Egyptian Code			D'Appolonia & Brissette			Burland & Burbidge		
Stage	Pre-tender	During -tender	Post-tender	Pre-tender	During-tender	Post-tender	Pre-tender	During-tender	Post-tender
First Building									
Settlement Values (mm)	108.5	92.1	154.8	74.4	67.0	90.3	47.2	41.0	67.2
Allowable settlement (mm)	100*			65**					
% more or less allowable***	+8.50	-7.90	+54.8	+14.50	+1.23	+38.9	-27.38	-36.92	+3.38
Second Building									
Settlement Values (mm)	82.6	66.9	116.4	55.2	48.4	64.7	39.7	33.6	54.7
Allowable settlement (mm)	70*			50**					
% more or less allowable***	+18.00	-4.43	+66.29	+10.40	-3.20	+29.40	-20.60	-32.80	+9.40
Third Building									
Settlement Values (mm)	49.6	40	69.8	33.1	28.2	37.9	25.1	20.5	34.4
Allowable settlement (mm)	70*			50**					
% more or less allowable***	-29.14	-42.86	-0.29	-33.80	-43.60	-24.20	-49.80	-59.00	-31.20

* According to EGC (2007). ** According to MacDonald and Skempton (1995)

*** *-ve* values are less than the allowable settlement limits, and *+ve* values are more.

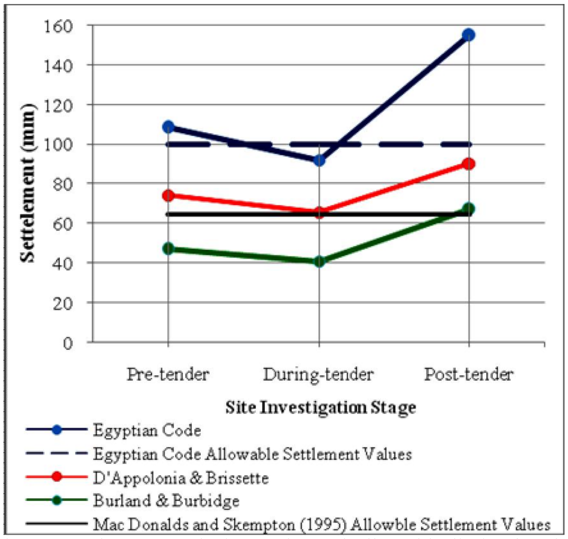

Fig. 7. Settlement analysis results and allowable limits for the first building.

From the above figure it could be observed that, using the different calculations methods the during-tender stage the settlement is always less than the allowable limits and according post-tender stage the settlement is always more than the allowable limits. While according to the pre-tender stage the settlement allowance is dependent on the calculation method.

4 CONCLUSION

The SPT can provide useful and reliable data with good maintenance of the equipment and quality control in the performance of the test. Using the same drilling crew and a good engineer on-site is important for the quality of the results.
The available field test results (SPT) for the three investigation stages show major variation between the results. The bearing capacity and settlement have been calculated for three different buildings in this research. The results show that the bearing capacity has variation in the safe margin for the three stages, but the settlement number was the predominate factor. The settlement has been calculated using different methods. The settlement results show large variation ranging from 10.0% to 73.0%. This large variation may affect the design safety and economy for the building. While, the settlement analysis results forced the designer to change the footing type, dimensions, system.

REFERENCES

Baecher G B. (1979). "Analysing exploration strategies." Site Characterisation and Exploration,

Baecher G B, and Christian J T. (2003). Reliability and Statistics in Geotechnical Engineering, John Wiley & Sons Ltd., Chichester, England.

Barton M.E.(1990), Field testing in engineering geology, Geological society special publication No6, pp. 121-127. Bowles, J.E. (1996), Foundation analysis and design, 5th edition, McGraw Hill International Editions.

Burland, J.B., Burbidge, M.C. (1984), Settlement of foundations on sand and gravel, Proceedings of the Institution of Civil Engineers, Part 1, 1985, 78, Dec., 1325-1381.

Duncan, J.M. (2000), Factors of Safety and Reliability in Geotechnical Engineering, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 126, No. 4, pp. 307-316

Filippas O B, Kulhawy F H, and Grigoriu M D. (1988). "Reliability-Based Foundation Design for Transmission Line Structures: Uncertainties in Soil Property Measurement." Research Report 1493-3 EL-5507, Vol. 3, Cornell University/ Electric Power Research Institute, Palo Alto, California, USA

Fletcher, G. (1965), Standard penetration test: Its uses and abuses, ASCE Journal of Soil Mechanics and Foundation Engineering, Vol. 91, SM-4, July, p.67.

Jaksa M B. (1995). "The Influence of Spatial Variability on the Geotechnical Design Properties of a Stiff, Overconsolidated Clay," PhD, The University of Adelaide, Adelaide.

Jones A.L., Kramer S.L., Arduino P., 2002. Estimation of Uncertainty in Geotechnical Properties for Performance-Based Earthquake Engineering. PEER 2002/16, Pacific Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley.

Kulhawy F H. (1992). "On Evaluation of Static Soil Properties." Stability & Performance of Slopes & Embankments II (GSP 31), R B Seed and R W Boulanger, eds., ASCE, New York, USA, pp. 95-115.

Kulhawy F H, and Mayne P W. (1990). "Manual on estimating soil properties for foundation design." EL-6800, Electric Power Research Institute, Palo Alto, California, USA.

Kulhawy F H, and Phoon K K. (2002). "Observations on Geotechnical Reliability-Based Design Development in North America." International Workshop on Foundation Design Codes and Soil Investigations in view of International Harmonisation and Performance Based Designs, pp. 31-48.

Kulhawy F.H, Trautmann, C.H.(1996), Estimation of in-situ test uncertainty, Proceedings of uncertainty 96: Uncertainty in the geologic environment: From theory to practice, pp.269-286.

Lumb P. (1974). "Application of Statistics in Soil Mechanics." Soil Mechanics - New Horizons, I Lee, ed., Butterworth and Co (Publishers), London, England, pp. 44-111.

Northwest University, Illinois, USA, pp. 220-246.

Vanmarcke E H. (1977a). "Probabilistic Modeling of Soil Profiles." Journal of the Geotechnical Engineering Division, 103(GT11), pp. 1227-1246.

Schmertmann, J.H. (1975), Measurement of insitu shear strength, keynote lecture, Proceedings of the conference on in-situ measurement of soil properties, June 1-4, 1975, vol. II, American Society of Civil Engineers.

Snedecor G W, and Cochran W G. (1980). Statistical Methods, The Iowa State University Press, Ames, USA.

Spry M J, Kulhawy F H, and Grigoriu M D. (1988). "Reliability-Based Foundation Design for Transmission Line Structures: Geotechnical Site Characterization Strategy." ResearchReport 1493-1 EL-5507, Vol. 1, Cornell University / Electric Power Research Institute, Palo Alto, California, USA.

Tavenas, F.A. (1971), Discussion to deMello (1971), 4th Panamerican Conference on Soil Mechanics and Foundation Engineering, Puerto Rico, Vol. III, pp. 64-69.

Vanmarcke E H. (1977b). "Reliability of Earth Slopes." Journal of the Geotechnical Engineering Division, 103(GT11), pp. 1247-1265.

Whitman R V. (2000). "Organising and Evaluating Uncertainty in Geotechnical Engineering." Journal of Geotechnical and Geoenvironmental Engineering, 126(7), pp. 583-593.

Youd, T.L., Idriss, I.M., Andrus, R.D., Arango, I., Castro, G., Christian, J.T., Dobry, R., Liam Finn, W.D., Harder, L.F.Jr., Hynes, M.E., Ishihara, K., Koester, J.P., Liao, S.S.C., Marcusson III, W.F., Martin, G.R., Mitchell, J.K., Moriwaki, Y., Power, M.S., Robertson, P.K., Seed, R.B., Stokoe II, K.H. (2001), Liquefaction resistance of soils, ASCE Journal of Geotechnical and Geoenvironmental Engineering, Vol. 127, No.10 October 2001.